addition, research is needed to determine necessary and/or sufficient convergence conditions with respect to the hill climbing random variable, $R_k(i, j)$, of the GHC algorithm; these conditions would provide valuable guidance when defining a $R_k(i, j)$ that will allow the GHC algorithm to solve the discrete optimization problem. Moreover, a convergence theory based on various hill climbing random variables may provide insight into a general convergence theory for a wide variety of GHC algorithm formulations.

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A Robust Smith Predictor Modified by Internal Models for Integrating Process with Dead Time

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***Abstract*—The internal model principle and control together (IMPACT) structure of the Smith predictor is proposed. The structure constructively achieves both the robust stability and absorption of external disturbances. In the structure design, the absorption principle is applied to enable the rejection of arbitrary class of deterministic disturbances and/or to suppress the effects of low frequency stochastic external signals on the system output. It is shown that the tuning of IMPACT structure is extremely simple due to relatively small number of tuning parameters all having clear physical meanings. The presented results of the simulation runs demonstrate the design procedure and illustrate the efficiency of the structure in disturbance absorption.**

***Index Terms*—Absorption principle, IMPACT structure, parameter tuning, robust stability, Smith predictor.**

I. INTRODUCTION

The concept of internal model consists in incorporation of the disturbance or/and plant models into the control portion of the system in order to suppress or even eliminate the effects of immeasurable external disturbances on the steady-state value of controlled variable and to increase the system robustness with respect to changes or uncertainties of plant parameters. Numerous papers have been published on the subject and they may be classified in two groups according to different approaches: internal model principle (IMP) includes the model or estimator of an external disturbance within the control section of the system [1]–[5], and internal model control (IMC) is based upon the inclusion of the nominal plant model into the system control structure [6]. The IMC approach has been advantageously practiced in the design of high-performance electrical drives and industrial processes.

Ya. Z. Tsypkin proposed the new control structure called internal model principle and control together (IMPACT) composed by the IMP and IMC [7]. It has been shown that many structures may be interpreted as using IMP and/or IMC of some kind. Some suggested are that of Tsypkin [8] with comparison to H^∞ and H^2 optimal controllers for controlling nonminimally phase control plants and with adaptation of the IMP portion of controller [9]. Tsypkin and Nadezhdin [10] utilized IMP for continuous-time control systems. The design of tracking systems with IMP and plants having a significantly long dead time has been proposed in [11]. Komada *et al.* proposed a new force control strategies based upon IMP that are robust against immeasurable torque disturbances and parameter variations of controlled electrical drives [12]. The sensitivity properties of the IMPACT structure with respect to measure noise were studied in [13]. See also the survey of IMP by Gonzales and Antsaklis [14]. Generally, the IMPACT structure excludes the effects of a known class of external disturbance on controlled variable and improves the system robustness with respect to changes of plant parameters.

It has been shown by Morary and Zafiriou [6], that the classical Smith predictor [15], which represents an effective compensator for a

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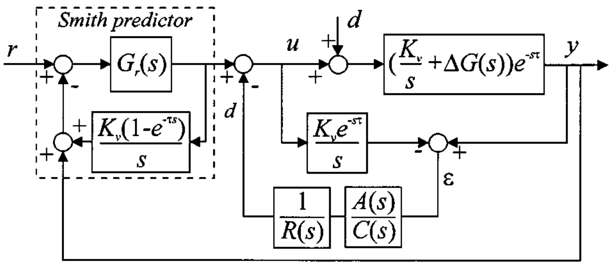

Fig. 1. IMPACT structure of the modified Smith predictor with one-input internal nominal plant model.

stable process with large dead time, was in the category of an IMC structure. Suitable modifications of Smith predictor have been proposed in [16]–[18] in order to enable an undelayed estimate of output and constructing of disturbance-compensating controller for a process with an integrator and long dead-time. In [18], simple criteria for tuning a dead-time compensator according to robust performance specifications for plants with an integrator mode and long dead-time are proposed. Different schemes of the Smith predictor modification and their comparison from the stand point of parameter setting and system robustness were presented in [19].

In this note, a new structure of the modified Smith predictor is proposed for processes that can be described by an integrator, a velocity gain and a long effective transport lag. The structure enables absorption of general class of disturbances and can be easily tuned to achieve the desired speed of set-point response and to maintain the preferred system robustness with respect to interval changes and/or uncertainties of plant parameters. The robustness analysis is given in detail and several simulations are presented to illustrate the design procedure and to confirm the structure ability in rejection of different classes of disturbance.

II. System Control Structure

The IMPACT control structure of the modified Smith predictor is shown in Fig. 1. For the analysis in this note, integrative industrial processes will be considered and described by the following transfer function:

$$G_p(s) = \frac{K_v}{s(T_1 s+1)(T_2 s+1)\cdots(T_n s+1)}\, e^{-\tau s} \tag{1}$$

where

- K_v velocity gain factor;
- τ process dead time;
- T_i $(T_i > 0,\ i = 1, 2, \ldots n)$ are process time constants.

The process transfer function may be rewritten as

$$G_p(s) = \left(\frac{K_v}{s} + \Delta G(s)\right) e^{-\tau s} \tag{2}$$

with

$$\Delta G(s) = \frac{k_1}{T_1 s+1} + \frac{k_2}{T_2 s+1} + \cdots + \frac{k_n}{T_n s+1} \tag{3}$$

where residues $k_i (i = 1, 2, \ldots, n)$ are functions of velocity gain K_v and process time constants T_i. For a long dead time, one can assume the nominal plant model as

$$G_{pn}(s) = \frac{K_v}{s}\, e^{-Ls} \tag{4}$$

where L is an identified effective transport lag and $\Delta G(s)$ in (2) is an unmodeled process dynamic. For the integrative plant (4), the proportional main controller $G_r(s) = K_r$ is applied.

The control portion within the system structure in Fig. 1 comprises the Smith predictor internal controller, in the main loop, and two internal models, in the local minor loop: the internal nominal plant model explicitly and the internal model of external disturbance $d(t)$ embodied in the transfer function $A(s)/C(s)$. Both the internal nominal plant model and disturbance model are treated as a disturbance estimator. Under the nominal case, the closed-loop transfer functions $y(s)/r(s)$ and $y(s)/d(s)$ are easily derived from Fig. 1 as

$$\frac{y(s)}{r(s)} = \frac{K_r K_v}{s + K_r K_v}\, e^{-Ls} \tag{5}$$

and

$$\frac{y(s)}{d(s)} = \frac{\left(1-\frac{1}{R(s)}\frac{A(s)}{C(s)}\frac{K_v}{s}e^{-Ls}\right)K_v\left[1+K_r\frac{K_v}{s}(1-e^{-Ls})\right]}{s+K_r K_v}\, e^{-Ls}. \tag{6}$$

In virtue of (5) and (6), the speed of set-point response can be adjusted by choosing appropriate values of controller gain K_r or dominant time constant $T_r = 1/K_r K_v$. Then, the absorption of an external disturbance, speed of disturbance transient response, and the system robustness with respect to uncertainties of plant parameters are adjusted by choosing the structure and parameters of the disturbance estimator.

Since term $(1 - e^{-Ls})/s$ in the numerator of closed-loop system transfer function (6) has the frequency characteristics of zero-order hold, the speed of disturbance transient response is governed by the roots of characteristic equation $(s + K_r K_v)C(s) = 0$. For example, if one chooses $C(s) = (T_o s + 1)^n$ (see Section III), the characteristic equation of disturbance estimator becomes

$$(s + K_r K_v)(T_o s + 1)^n = 0. \tag{7}$$

Thus, choosing proper values of n and of tuning parameter T_o, one can settle the speed of disturbance absorption. In doing so, lower order n and smaller value of T_o will correspond to a faster rejection of disturbance and a lower degree of system robustness, and vice versa. For the sake of simplicity and easier physical realization, it is usually assumed $n = 2$. The results of simulation runs, given later in this note, show that, with $n = 2$, notable system robustness is attained. Hence, the main feature of the IMPACT structure in Fig. 1, consists in extremely simple and straightforward adjustments of the set-point transient response, speed of absorption of an expected class of disturbance, and degree of system robustness. This is accomplished independently; first by choice of an appropriate value of T_r and then by setting of tuning parameter T_o.

Notice, due to the presence of integration mode in the internal one-input nominal plant model, the structure of Fig. 1 is internally unstable. This obstacle may be overcome by transforming the structure of Fig. 1 into the equivalent one having the internal two-input nominal plant model, shown in Fig. 2, which is internally stable. In the structure of Fig. 2, polynomial $C(s) = (T_o s + 1)^n$ is chosen.

III. Principle of Absorption

From (6), the steady-state error in the presence of a known class of external disturbance $d(t)$ will become zero if

$$\lim_{s \to 0} s \cdot \frac{\left(1-\frac{1}{R(s)}\frac{A(s)}{C(s)}\frac{K_v}{s}e^{-Ls}\right)K_v\left[1+K_r\frac{K_v}{s}(1-e^{-Ls})\right]}{s+K_r K_v} \cdot e^{-Ls}\, d(s) = 0 \tag{8}$$

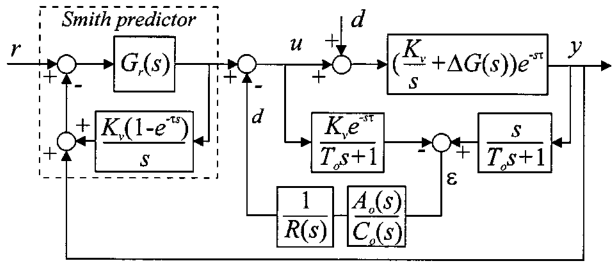

Fig. 2. Internally stable IMPACT structure of the modified Smith predictor with two-input internal nominal plant model.

In the case of integrative plant, the proper choice of $R(s)$ is $R(s) = K_v$. Since

$$\lim_{s \to 0} \frac{K_v \left[1 + K_r \dfrac{K_v}{s}\left(1 - e^{-Ls}\right)\right]}{s + K_r K_v} e^{-Ls} \neq 0 \tag{9}$$

the relation (8), for $R(s) = K_v$, is reduced to

$$\lim_{s \to 0.} s\left[1 - \frac{A(s)}{sC(s)} e^{-Ls}\right] d(s) = 0 \tag{10}$$

As it will be shown later, the stable polynomial $C(s)$ is to be chosen according to the desired speed of disturbance transient response and degree of system robustness and then polynomial $A(s)$ is determined so to satisfy relation (10). Due to stability conditions, polynomial $A(s)$ must contain a single zero at the origin and thus it may be replaced by $A(s) = sA_0(s)$, $A_0(0) \neq 0$. With stable polynomial $C(s)$ and $A(s) = sA_0(s)$, relation (10) may be rewritten as

$$\lim_{s \to 0.} s\left[C(s) - A_0(s)e^{-Ls}\right] d(s) = 0 \tag{11}$$

For chosen stable polynomial $C(s)$ and the class of polynomial disturbances $d(t) = \sum_{i=1}^{m} d_i t^{i-1}$, polynomial $A(s)$ is uniquely determined from (11) by calculating

$$\lim_{s \to 0} \frac{d^k}{ds^k}\left[C(s) - A_0(s)e^{-Ls}\right] = 0, \qquad 0 \le k < m \tag{12}$$

where, for example, for the constant, ramp, and parabolic disturbances [or for $d(s) = 1/s$, $d(s) = 1/s^2$, and $d(s) = 1/s^3$], $k = 0$, $k = 0$ and 1, and $k = 0$, 1, 2, and so on.

However, most frequently disturbances may be considered as slow varying and in these cases the polynomial $A(s)$ should be calculated to correspond to the ramp signal $d(t)$. As the experimental results given later in this note will show, polynomial $A(s)$ that corresponds to a ramp disturbance efficiently absorbs constant, ramp, and slow varying disturbances and even it suppresses the effects of low frequency stochastic external signals.

Hence, in the design of local minor loop inside the control structure of Fig. 1 it is first necessary to chose polynomial $C(s)$. This can be done according to the desired speed of disturbance response and degree of system robustness with respect to uncertainties of plant parameters K_v and L.

For the sake of clarity and to reduce the number of adjustable parameters, let us assume $C(s) = (T_o s + 1)^n$. Then, for rejection of ramp disturbance, relation (12) gives

$$A_0(0) = 1, \qquad \text{for } k = 0 \tag{13a}$$

$$\frac{dA_0(s)}{ds} = nT_0 + L$$

or

$$A_0(s) = A_0(0) + s(nT_0 + L), \qquad \text{for } k = 1 \tag{13b}$$

wherefrom one calculates $A_0(s) = s(nT_0 + L) + 1$ and the transfer function inside the disturbance estimator becomes

$$\frac{1}{R(s)} \frac{A(s)}{C(s)} = \frac{1}{K_v} \frac{s\left[s(nT_o + L) + 1\right]}{(T_o s + 1)^n}. \tag{14}$$

The control part of IMPACT structure of modified Smith predictor in Fig. 1 contains five parameters K_v, L, K_r, T_o, and n. Two of them, plant parameters K_v and L, are measured or estimated by simple experiment. Other three parameters K_r, T_o, and n are to be adjusted with respect to prescribed speeds of set-point transient and disturbance transient responses and to the desired degree of system robustness with respect to mismatches of K_v and L.

Moreover, it is possible to design the observer estimator that rejects any kind of expected disturbance. To this end, suppose the class of disturbances having the Laplace transform $d(s) = N(s)/D(s)$. Then, relation (11) is satisfied if the following equation holds:

$$C(s) - A_0(s)e^{-Ls} = \Phi(s)B(s) \tag{15}$$

where $\Phi(s)$ represents the so-called absorption polynomial determined by $\Phi(s) \equiv D(s)$. For example, to the polynomial and sinusoidal disturbances ($d(t) = \sum_{i=1}^{m} d_i t^{i-1}$ and $d(t) = \sin \omega t$) correspond $\Phi(s) = s^{m+1}$ and $\Phi(s) = s^2 + \omega^2$, respectively.

To reduce (15) into polynomial equation, the exponential term e^{-Ls} is approximated by the finite power series

$$e^{-Ls} = \sum_{k=0}^{N} \frac{(-Ls)^k}{k!}. \tag{16}$$

Substituting e^{-Ls} from (16) into (15), relation (15) obtains the specific form of the Diophantine equation

$$A_0(s) \sum_{k=0}^{N} \frac{(-Ls)^k}{k!} + B(s)\Phi(s) = C(s). \tag{17}$$

A single solution of the Diophantine equation, which plays a crucial role in the design procedure of the observer estimator, proposed in this note, does not exist [20]. Relation (17) is a linear equation in polynomials $A_0(s)$ and $B(s)$. Generally, the existence of the solution of the Diophantine equation is given in [21]. According to [21], there always exists the solution of (17) for $A_0(s)$ and $B(s)$ if greatest common factor of polynomials $\sum_{k=0}^{N}(((-Ls)^k)/k!)$ and $\Phi(s)$ divides polynomial $C(s)$; then, the equation has many solutions. The particular solution is constrained by the fact that the control law must be causal, i.e., $\deg A(s) = 1 + \deg A_0(s) \le \deg C(s)$. Hence, after choosing a stable polynomial $C(s)$, N, and degrees of polynomials $A_0(s)$ and $B(s)$, and inserting the absorption polynomial $\Phi(s)$ that corresponds to an expected external disturbance, polynomials $A_0(s)$ and $B(s)$ are calculated be equating coefficients of equal order from the left- and right-hand sides of (17).

IV. Robustness Analysis

Linear continuous models of finite orders fairly well approximate dynamic behavior of plants at a low frequency range; disagreements appear at high frequencies [6]. Differences between the nominal plant $G_{pn}(s)$ and real plant $G_p(s)$ appear due to unmodeled dynamics and uncertainties and/or perturbations of plant parameters. In the robustness analysis, real plant $G_p(s)$ is considered as a member of the infinite family of plants within which each member more or less deviates from the nominal plant $G_{pn}(s)$. Thus, the family describes all plants and may be written as

$$\mathrm{P} = \left\{G_p\colon\ |G_p(j\omega) - G_{pn}(j\omega)| \le \bar{l}_a(\omega)\right\} \tag{18}$$

where $\bar{l}_a(\omega)$ represents the additive bound of uncertainty. Note that, for the same purpose, the so-called multiplicative bound of uncertainty is used [6], [7]. Hence, each member of the family satisfies the relation

$$G_p(j\omega) = G_{pn}(j\omega) + l_a(j\omega) \tag{19}$$

with $|l_a(j\omega)| \le \bar{l}_a(\omega)$.

Suppose that $G_p(s)$ and $G_{pn}(s)$ have the same number of unstable poles (in the right half-plane) and that the desired closed-loop system transfer function $G_m(s)$ is stable. Then, each member of the family is stable if and only if the following criterion of robust stability is satisfied [6]: $\bar{l}_a(\omega) < \beta(\omega)$, where

$$\beta(\omega) = \left|\frac{G_{pn}(j\omega)}{G_m(j\omega)}\right| \left|\frac{G_{ff}(j\omega)}{G_{fb}(j\omega)}\right| \tag{20}$$

while $G_{ff}(s)$ and $G_{fb}(s)$ are defined from

$$u(s) = G_{ff}(s)r(s) - G_{fb}(s)y(s) \tag{21}$$

as the transfer functions of feedforward and feedback portions of the system control structure, respectively.

For the IMPACT structure of Fig. 2, one derives

$$\beta(\omega) = K_v \left|\frac{T_r j\omega + 1}{j\omega}\right| \cdot \left|\frac{C(j\omega)}{C(j\omega) + (T_r j\omega + 1 - e^{-Lj\omega})A_0(j\omega)}\right|. \tag{22}$$

From the above analysis, one can conclude that the design of minor local loop of the IMPACT structure may contribute to the system robustness, but only for given interval changes and/or uncertainties of plant parameters.

Notice from (22) that $\beta(\omega)$ tends to a constant value at high frequencies. Namely, if one chooses $C(s) = (T_0 s + 1)^n$ and $A_0(s) = a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0$, then

$$\lim_{\omega \to \infty} \beta(\omega) = \frac{K_v T_r T_0^n}{T_0^n + T_r a_{n-1}}, \quad \text{for } \deg C(s) = \deg A(s) (= 1 + \deg A_0(s)) \tag{23a}$$

$$\lim_{\omega \to \infty} \beta(\omega) = K_v T_r, \qquad \text{for } \deg C(s) > \deg A(s) \tag{23b}$$

It is evident from (22) that a greater value of $T_r = 1/K_r K_v$ yields a higher degree of system robustness. Consequently, to improve the system robustness, the speed of set-point response must be slowed down. The influence of disturbance estimator on system robustness will be investigated by the illustrative example in the section that follows.

V. Simulation Results

First, we shall investigate the influence of disturbance estimator on system robustness. To this end, let us consider particular example of the process given by [19]

$$G_p(s) = \frac{0.1e^{-8s}}{s(1+s)(1+0.5s)(1+0.1s)} \tag{24}$$

with identified nominal plant model

$$G_{pn}(s) = \frac{0.1e^{-9.7s}}{s}. \tag{25}$$

In the example, the disturbance observer is designed to absorb ramp disturbances. Thus, by setting pertinent value $T_r = 2$, and $K_v = 0.1$,

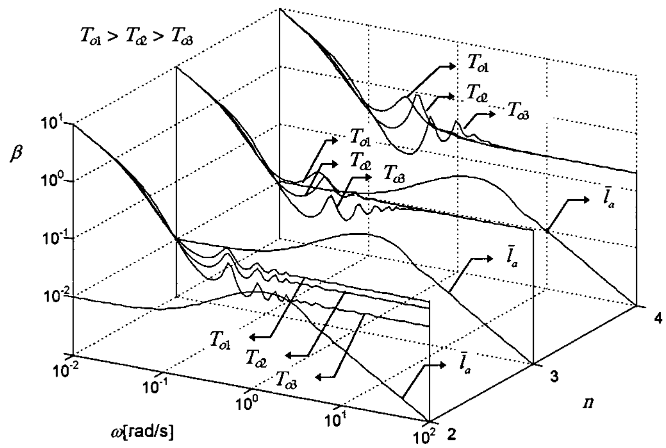

Fig. 3. Influence of disturbance estimator parameters on the robust stability.

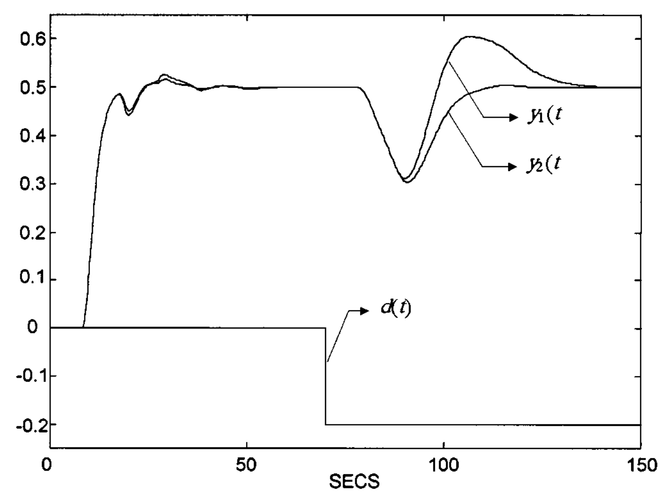

Fig. 4. The absorption of a constant disturbance.

$C(j\omega) = (T_0 j\omega + 1)^n$, and $A_0(j\omega) = (nT_0 + 9.7)j\omega + 1$ into (22), one obtains

$$\beta(\omega) = 0.1 \left|\frac{2j\omega + 1}{j\omega}\right| \cdot \left|\frac{(T_0 j\omega + 1)^n}{(T_0 j\omega + 1)^n + (2j\omega + 1 - e^{-9.7j\omega})\,[(nT_0 + 9.7)j\omega + 1]}\right|. \tag{26}$$

In Fig. 3, the additive bound of uncertainties $\bar{l}_a(\omega) = |G_p(j\omega) - G_{pn}(j\omega)|$ is shown three times together with $\beta(\omega)$ drawn for $n = 2$, 3, and 4 and for different values of T_0: $(T_{01}, T_{02}, T_{03}) = (9, 6, 3)$. In virtue of Fig. 3, for a higher degree n of chosen polynomial $C(s)$ and a greater value of time constant T_0, the system robustness improves. In other words, lower speed of disturbance rejection corresponds to the growth of system robustness, and vice versa. Recall that, in the example, the observer estimator is designed to absorb ramp disturbances. The same IMPACT structure with observer estimator

$$\frac{1}{R(s)} \frac{A(s)}{C(s)} = \frac{1}{K_v} \frac{s}{(T_o s + 1)^n} \tag{27}$$

designed to reject constant disturbances will exhibit better robustness. Generally, the design of the local minor loop for absorption of more complex external disturbances requires a higher order of polynomial $A_0(s)$ and, according to (22), results in a lower degree of robustness.

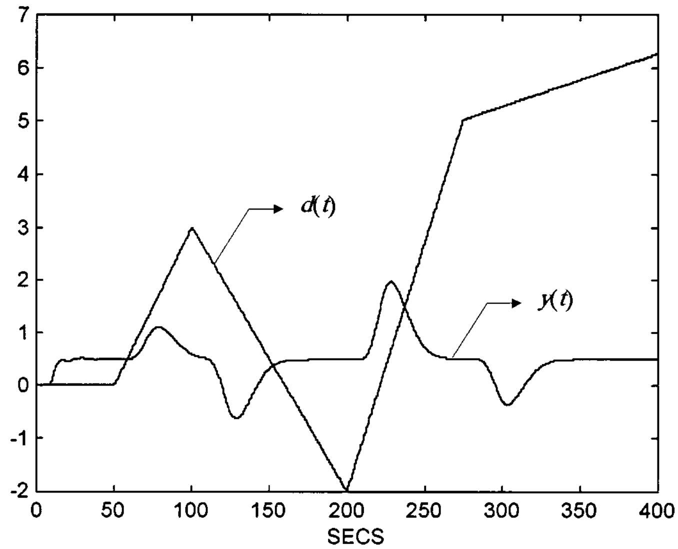

Fig. 5. The absorption of a combined ramp disturbance.

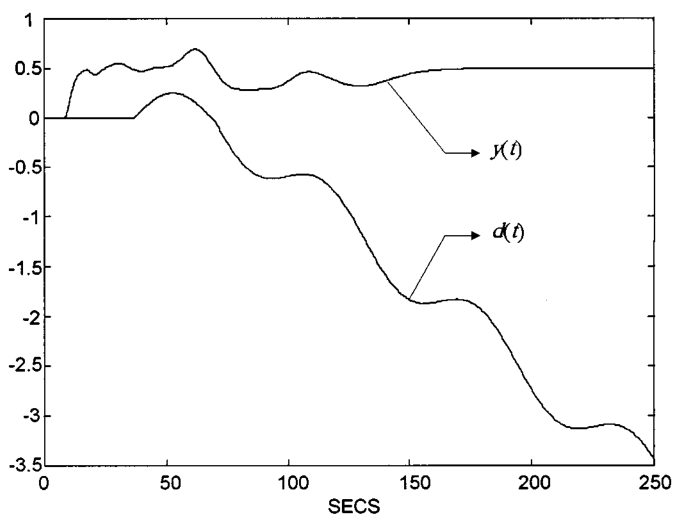

Fig. 6. The absorption of a more complex disturbance.

To illustrate the efficiency of the IMPACT structure (Fig. 2) in disturbance absorption, we consider the example of control plant given by (24) and (25). In all simulation runs the reference is $r(t) = 0.5 \cdot 1(t)$ and $T_r = 2$. Fig. 4 explains by example the absorption of constant disturbance $d(t) = -0.2 \cdot 1(t - 70)$. First, the structure in Fig. 2 is designed to absorb a ramp disturbance by using transfer function (14), with $n = 2$ and $T_0 = 1$. Trace $y_1(t)$ of Fig. 4 shows the disturbance response. Second, the structure is designed to absorb a constant disturbance using (27), with $n = 2$ and $T_0 = 1$, and trace $y_2(t)$ is obtained. In both cases, the constant disturbance is absorbed during the transient and consequently does not affect the steady-state value of the output.

In the second example, the combined ramp disturbance shown in Fig. 5 is applied. The structure in Fig. 2 is designed to absorb ramp disturbances by transfer function (14), with $n = 2$ and $T_0 = 6$. Fig. 5 illustrates the disturbance absorption. Notice from the figure that each linear segment of the disturbance is absorbed after certain time period.

To emphasize the capability of the proposed IMPACT structure of Fig. 2 for the absorption of an arbitrary class of disturbances, the more complex disturbance $d(t) = 0.25\ \sin(0.1(t - 37)) \cdot 1(t - 37) - 0.02(t - 70) \cdot 1(t - 70)$ is applied. To this disturbance corresponds

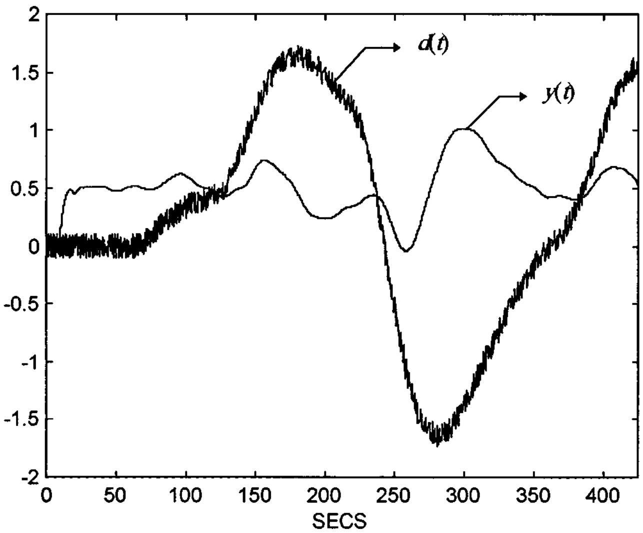

Fig. 7. The absorption of a noise contaminated slow varying disturbance.

absorption polynomial $\Phi(s) = s^2(s^2 + 0.1^2)$. Substituting $\Phi(s)$ into (17) and choosing $N = 4$ and $C(s) = (6s + 1)^5$, the Diophantine equation (17) is solved for $A_0(s)$, using the procedure outlined in the last paragraph of Section III, to obtain $A(s) = sA_0(s) = s(5057s^3 + 287.5s^2 + 39.7s + 1)$. Fig. 6 illustrates the process of absorption during the transient and complete rejection of disturbance in the steady state.

Most frequently disturbances are slow varying. Such a disturbance contaminated with the white noise of variance 0.1 is shown in Fig. 7. To insure its rejection, the structure in Fig. 2 is designed to absorb ramp disturbances by using (14), with $n = 2$ and $T_0 = 6$. Fig. 7 illustrates the disturbance absorption and suppression of noise contamination. The disturbance rejection may be further improved by choosing $n = 1$ and/or a smaller value of T_0. However, in doing so, one must maintain the robust stability with respect to uncertainties of plant parameters.

VI. Concluding Remarks

We have proposed a new structure of the Smith predictor for control plants with the integration mode, velocity constant, process time constants, and relatively long transport lag. It is to be noted that the similar design procedure can be carried out in the case of static plants by including the I-action into the main controller. The structure comprises the Smith controller and two internal models: the two-input nominal plant model explicitly and model of disturbance embodied into the disturbance estimator. The structure is internally stable and may be adjusted, according to the desired speed of set-point response and speed of disturbance rejection, in a simple way by tuning only three parameters having clear physical meanings. The observer estimator is designed by the absorption principle, which enables the structure to absorb any class of disturbances, arbitrary slow varying disturbances, and low frequency stochastic external signals. It has been shown that the structure design is possible for interval uncertainties of plant parameters. This constraint can be taken into account by the additive bound of uncertainty and the criterion of robust stability, employed in this note. Several simulation results are presented to illustrate the design procedure and to demonstrate the efficiency of the structure in disturbance rejection.

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Power Characterizations of Input-to-State Stability and Integral Input-to-State Stability

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***Abstract*—New notions of external stability for nonlinear systems are introduced, making use of average powers as signal norms and comparison functions as in the input-to-state stability (ISS) framework. Several new characterizations of ISS and integral ISS are presented in terms of the new notions. An example is discussed to illustrate differences and similarities of the newly introduced properties.**

***Index Terms*—Continuous-time, inputs, nonlinear, power, stability.**

I. INTRODUCTION

The notion of input-to-state stability (ISS) has been now widely recognized and accepted as an important concept that is useful for a range of nonlinear control problems (see [4]–[6]). Since its conception in [11] a range of important equivalent characterizations of ISS have been proved in the literature (see [9]–[13]). These characterizations lead to a better understanding of the ISS property and provided the control engineers with a range of new tools that can be used in nonlinear control. ISS has been originally defined in $\mathcal{L}_\infty$ framework (see [11]) and it requires roughly speaking that "no matter what the initial state is, if the inputs are uniformly small, then the state must eventually be small." Results in [10] have shown that ISS systems also posses the property that bounded energy inputs imply bounded energy states. On one hand, this has shown that the ISS concept is more general than originally thought since it also covers L_2 stable systems. On the other hand, this research has lead to the introduction of a new property, the so called integral input-to-state stability property (iISS), which requires an ISS-like estimate for the solutions where the $\mathcal{L}_\infty$ norm of inputs is replaced by some energy function. The iISS property has been shown to be a natural generalization of ISS and it is anticipated that it will be at least as useful as ISS in the analysis of nonlinear control systems. A range of equivalent characterizations of iISS have been presented in [2], [3], [7], and [10], and they serve to better understand the property itself and to provide new tools that may be useful in different situations.

The purpose of this note is to provide new definitions of ISS-like and iISS-like properties. These are given in terms of powers of input and/or state signals and are novel compared to previous characterizations since they describe the system's behavior for different classes of input signals. Since bounded power signals may fail to be uniformly bounded or may have unbounded energy, for such signals power estimates might turn out to be tighter than the ones provided by the original ISS and iISS definitions. On the other hand, bounds expressed in terms of averaged signals cannot be translated into hard bounds on pointwise signal norms without some careful handling. Nevertheless, it is somewhat surprising that both ISS and iISS properties do have equivalent "power" characterizations. For instance, under a mild assumption of local stability we show that ISS is equivalent to the property that "no matter what the initial state is, if the *power* of inputs is uniformly small, then the *power* of state must eventually be small."

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